"I met Bernstein."

– Oleg Ivrii, facebook conversation.

The Trivial Notions Seminar
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The Bernstein-Kushnirenko theorem and other links between polytopes and algebraic geometry

A talk by

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Abstract

What can be said about the number of solutions of a system of k polynomial equations in n variables? The well-known Bézout theorem gives an answer in $\mathbb{CP}^n$. The Bernstein-Kushnirenko theorem (1975) expresses the exact number of solutions in $(\mathbb{C}\backslash\{0\})^n$ counted with multiplicities for the generic system in terms of the volumes of *Newton polytopes* – the convex hull in $\mathbb{R}^n$ of the degree vectors of the monomials in the equation. The formula is surprisingly simple, which suggests more connections between algebraic geometry and Newton polytopes.

Later, A. Khovanskii used the theory of toric varieties to develop such connections. For example, it turns out that the Ehrhart formula – the polynomial dependence of the number of integer points inside the polytope $k\Delta$ on the scaling factor k – is a corollary of the Riemann-Roch theorem.

During the talk we will prove the Bernstein-Kushnirenko theorem and discuss the relation between the Riemann-Roch theorem and Ehrhart polynomials. We will see more examples of such links and far-going generalizations of the theory.

Thursday, November 6$^{\text{th}}$, at 1:15 pm
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