THE TRIVIAL NOTIONS SEMINAR

Natalie Stewart

will speak on

From $\mathbb{Q}$ to $\mathbb{R}$: A leisurely review of Cauchy completion

ABSTRACT

For $\mathcal{V}$ a fixed Bénabou cosmos, we review the construction of the $\mathcal{V}\mathbf{Cat}$-enriched categories $\underline{\mathcal{V}\mathbf{Cat}} \subset \underline{\mathcal{V}\mathbf{Prof}}$ of small $\mathcal{V}$-enriched categories and functors or profunctors, respectively. For $\mathcal{C}$ a $\mathcal{V}$-category, we go on to define the *Cauchy completion* $\overline{\mathcal{C}} \subset \underline{\mathcal{V}\mathbf{Prof}}(I, \mathcal{C})$ to be the full $\mathcal{V}$-subcategory of $\mathcal{V}$-profunctors admitting a right adjoint. For $\mathcal{V} = ([0, \infty], \geq, +)$, this recovers the Cauchy completion of Lawvere metric spaces, and after symmetrization, this recovers the Cauchy completion of ordinary metric spaces.

We define a type of weighted colimit, called an *absolute colimit*, which categorifies the property that all short maps preserve limits of Cauchy sequences. We state a universal property realizing the Cauchy completion as the *absolute cocompletion*, recovering the traditional definition of Cauchy complete metric spaces and the traditional universal property satisfied by the Cauchy completion of a metric space.

Time-permitting, we go on to characterize the Cauchy completion of categories, which is computed by the idempotent-splitting completion. We further go on to characterize the Cauchy completion of a ring as a preadditive category, which is computed by the category of finitely generated projective modules.

Tuesday, February 8, 2022

at 1:30 pm

Science Center, Room 232